Summary of the convergence tests that may appear on the Calculus BC exam.

Test Name	The series ...	will converge if	Or will diverge if	Comments
n^{th} –term test	$\sum_{n=1}^{\infty} a_n$		$\lim_{n\to\infty} a_n \neq 0$	For divergence only; the converse is false.
Geometric series	$\sum_{n=1}^{\infty} ar^{n-1}$	$-1 < r < 1$	$r \leq -1$ or $r \geq 1$	Sum = $\frac{a}{1-r}$
Alternating series test	$\sum_{n=1}^{\infty} (-1)^{n-1} a_n$	$\lvert a_{n+1}\rvert < \lvert a_n\rvert$ and $\lim_{n\to\infty} a_n = 0$		Error bound $\lvert S_\infty - S_n\rvert < \lvert a_{n+1}\rvert$
Integral test	$\sum_{n=1}^{\infty} a_n$ and $a_n = f(n) \geq 0$	$\int_1^{\infty} f(x)\,dx$ converges	$\int_1^{\infty} f(x)\,dx$ diverges	*f* must be continuous, positive and decreasing.
***p*-series**	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	$p > 1$	$p \leq 1$	
Direct comparison test	$\sum_{n=1}^{\infty} a_n$	$0 < a_n \leq b_n$ and $\sum_{n=1}^{\infty} b_n$ converges	$0 < b_n \leq a_n$ and $\sum_{n=1}^{\infty} b_n$ diverges	
Limit comparison test	$\sum_{n=1}^{\infty} a_n$	$a_n > 0, b_n > 0$ $\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$ and $\sum_{n=1}^{\infty} b_n$ converges	$a_n > 0, b_n > 0$ $\lim_{n\to\infty} \frac{a_n}{b_n} = L > 0$ and $\sum_{n=1}^{\infty} b_n$ diverges	See note below
Absolute convergence implies convergence	$\sum_{n=1}^{\infty} a_n$	$\sum_{n=1}^{\infty} \lvert a_n\rvert$ converges	If $\sum_{n=1}^{\infty} \lvert a_n\rvert$ diverges, $\sum_{n=1}^{\infty} a_n$ may still converge. (Conditional convergence).	
Ratio test	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \frac{\lvert a_{n+1}\rvert}{\lvert a_n\rvert} < 1$	$\lim_{n\to\infty} \frac{\lvert a_{n+1}\rvert}{\lvert a_n\rvert} > 1$	If $\lim_{n\to\infty} \frac{\lvert a_{n+1}\rvert}{\lvert a_n\rvert} = 1$ the ratio test cannot be used.

Another useful convergence tests that may be used, but not tested:

Test Name	The series ...	will converge if	Or will diverge if	Comments
Root Test	$\sum_{n=1}^{\infty} a_n$	$\lim_{n\to\infty} \sqrt[n]{a_n} < 1$	$\lim_{n\to\infty} \sqrt[n]{a_n} > 1$	The test cannot be used if $\lim_{n\to\infty} \sqrt[n]{a_n} = 1$

If one test is inconclusive, try a different test. There is no single test that must be used on a given series.

For the **Limit Comparison Test**

$$\text{If } L = 0 \ \& \ \sum_{n=1}^{\infty} b_n \text{ converges} \Rightarrow \sum_{n=1}^{\infty} a_n \text{ converges;}$$

$$\text{If } L = \infty \ \& \ \sum_{n=1}^{\infty} a_n \text{ converges} \Rightarrow \sum_{n=1}^{\infty} b_n \text{ converges.}$$