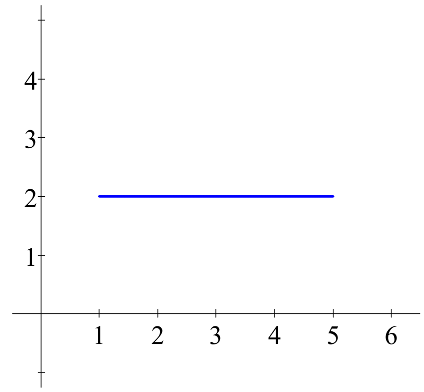
A Different Approach to Average Value

How do you average an infinite number of numbers?

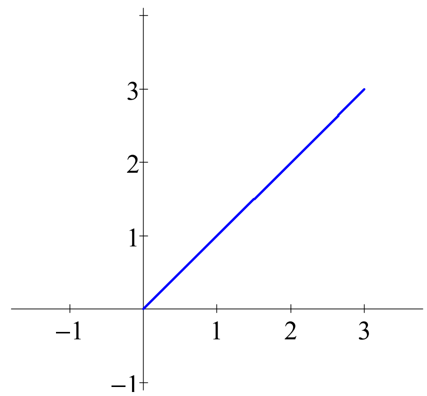
1. Consider the function $f(x) = 2$ for $1 \le x \le 5$.



What are the *y*-coordinates of the points of this function?

What is their average? Why?

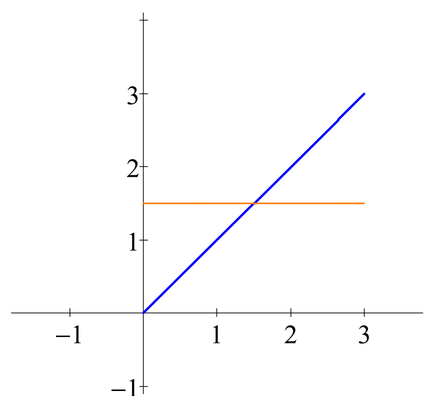
2. Now consider the function $y = x$ for $0 \le x \le 3$. The *y*-coordinates go quite evenly from 0 to 3.



a. What is the average of all the *y*-coordinates? ______

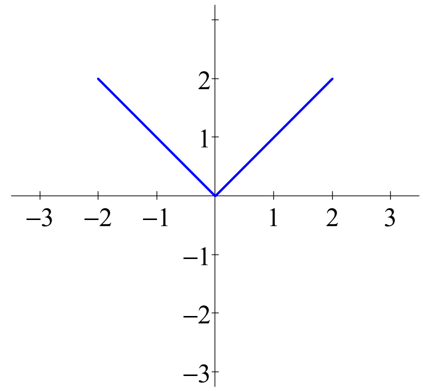
Explain why you think this is so?

b. The graph shows the function and a horizontal segment at $y = \frac{3}{2}$. Does it seem reasonable that the *y*-coordinates of the points on $y = x$ above this segment will “balance” the y-coordinates below the segment?



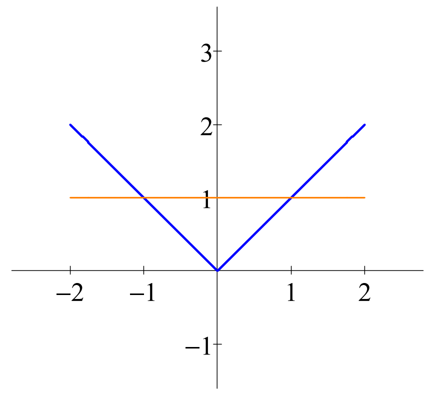
c. Find the area of the region between $y = x$ and the *x*-axis between 0 and 3. How does it compare to the area between the horizontal segment and the *x*-axis?

3. Now consider the function $y=|x|$ for $-2 \le x \le 2$.



a. What appears to be the average of the *y*-coordinates of this function?

b. In the next drawing there is a horizontal segment drawn at $y=1$. Does this segment seem to "balance" the *y*-coordinates of the function above and below it?

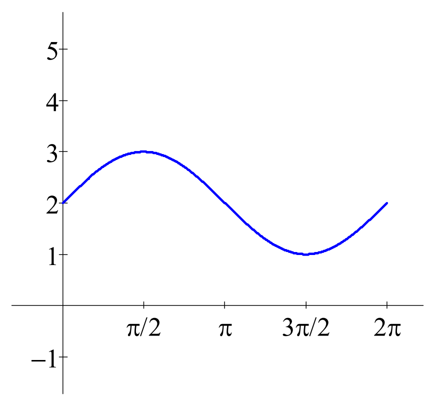


c. How do the areas of the regions between the function and the *x*-axis and between the horizontal segment and the *x*-axis compare?

4. Next consider $y=2+\sin(x)$ for $0 \le x \le 2\pi$.

a. What seems to be the average value of all the *y*-coordinates? ________ Draw a horizontal line at this value.

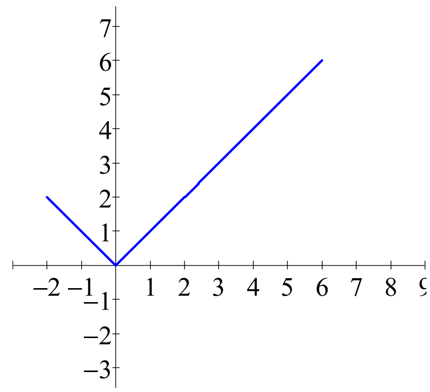
b. Do the *y*-coordinates above and below this value seem to "balance out"?



c. Find the area between $y=2+\sin(x)$ for $0 \le x \le 2\pi$ and the *x*-axis? (Show your work)

d. Is your answer to c the same as the area under the average value line on this interval?

5. (Be careful!) This time the function is $y = |x|$ for $-1 \le x \le 6$

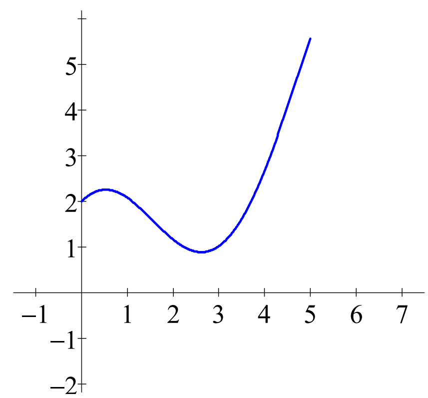


a. What appears to be the average value? __________

b. Draw a horizontal segment at the average value. Does the amount of the graph above and below seem to balance? How about the areas? Are they the same?

c. If things are not working as before, try using the area between the function and the *x*-axis to find the average value.

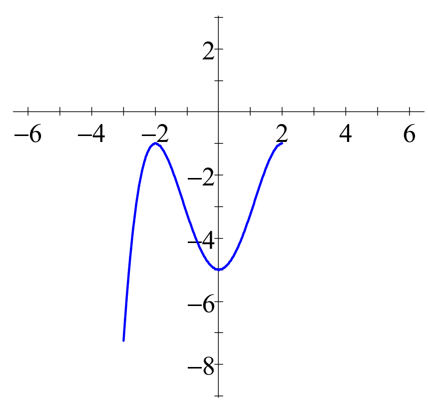
6. Consider the function $y = x + 2\cos(x)$ for $0 \le x \le 5$.



a. Use the area idea to find the average value.

b. Draw a horizontal line as before. Does the horizontal line look like it's in the right place?

7. Consider the function $y = -\frac{1}{4}x^4 + 2x^2 - 5$ for $-3 \le x \le 2$.

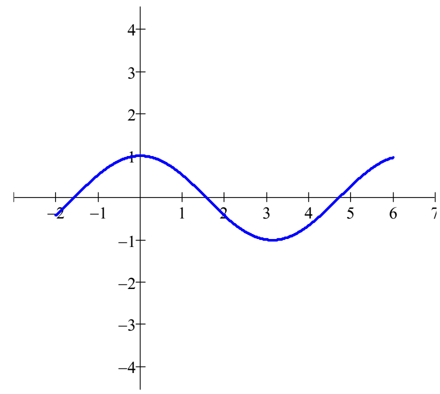


a. Use the area of the region between the graph and the *x*-axis to find the average value.

b. Draw a horizontal line as above. Does the horizontal line look like it's in the right place?

c. Your answer should be negative. Why?

8. Consider the function $y = \cos(x)$ for $-2 \le x \le 6$



a. Use the area idea to find the average value.

b. Draw a horizontal line as above. Does the horizontal line look like it's in the right place?

c. Part of the region here has a "negative area." How does that affect finding the average value?

9. Summarize your results with a formula that will give you the average value (average of the *y*-coordinates) of a function $f(x)$ on an interval $a \le x \le b$.

Average value = ________________________________

Numerical answers:

1a. 2, b. 2
2a. 3/2 = 1.5, b. yes, c. Areas are equal
3a. 1, b. yes, c Areas are equal
4a. 2, b. yes, c. 4π , d. yes
5a. 5/2 = 2.5, b. yes, c. 1(2) + 3(6) = 20, $\frac{20}{8} = 2.5$ A weighted average should be used here.
6a. 2.116, b. yes
7a. $\frac{-15.416}{5} = -3.083$, b. yes
8a $\frac{0.62988}{8} = 0.079$
9. $\bar{y} = \frac{\text{area}}{\text{length of interval}} = \frac{\int_a^b f(x)\,dx}{b-a}$

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