Half a trapezoid is better than …

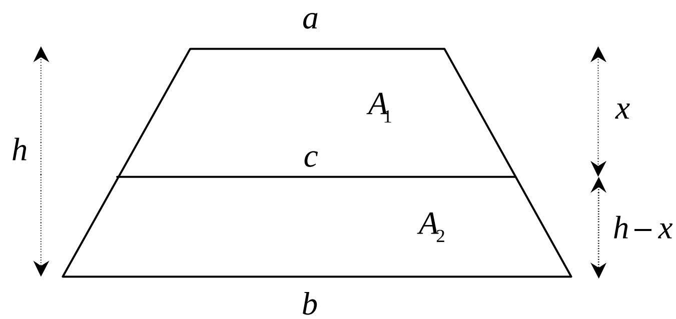
From Dr. Jing 3-11-07

A trapezoid with base 1 = a, and base 2 = b. Draw a segment that is parallel to the bases and divides the trapezoid's area A into A1 and A2. Represent the length of the segment in terms of a and b if A1 = A2.

Dr. Jing
St. Anthony High School
Long Beach, CA

My solution:

Let $0 < a < c < b$, A = area of original trapezoid, and let h = height of the original trapezoid (bases a and b) and let x = height* of trapezoid with bases a and c



Then equating the areas gives:

$$A_1 = \tfrac{1}{2}A$$
$$A_1 = A_2$$

or

$$\tfrac{1}{2}(a+c)x = \tfrac{1}{2}\tfrac{1}{2}(a+b)h$$

and

$$\tfrac{1}{2}(a+c)x = \tfrac{1}{2}(c+b)(h-x)$$

Solving this system for c and x with a CAS gives

$$c = \frac{\sqrt{2\left(a^2+b^2\right)}}{2} = \sqrt{\frac{a^2+b^2}{2}}$$

$$x = \frac{2a-\sqrt{2\left(a^2+b^2\right)}}{2(a-b)}h = \left(\frac{a-c}{a-b}\right)h$$

Dr. Jing's solution:

In a message dated 3/12/2007 6:49:14 A.M. Eastern Daylight Time, sahsjing@yahoo.com writes:

Thanks for sharing your idea.

My approach might be little bit different. If I use your notation, I have
(1/2)(a+b)h = (b+c)x……(1), total area of the trapezoid is twice the area of the trapezoid at the bottom.

(c-a)/(b-a) = (h-x)/h……(2), ratio of corresponding sides of two similar triangles is in proportion.

Solve (2) for x/h,
x/h = (b-c)/(b-a)……(3)
Substitute (3) in (1) and solve for c,
c = sqrt[0.5(a^2+b^2)]

And Gray Litvin does it this way:

In a message dated 3/12/2007 6:28:57 P.M. Eastern Daylight Time, litvin@skylit.com writes:
A slightly different approach:

Suppose the extensions of the sides of the trapezoid intersect at P.
These lines form three similar triangles with bases a, b, c
respectively and the third vertex at P.
The ratio of the areas of similar triangles is equal to the squared ratio
of the respective sides. If the area of the triangle with base c is A
then (A - A1)/A = a^2/c^2 and (A+A2)/A = b^2/c^2.
Adding together and considering that A1 = A2 we get
a^2/c^2 + b^2/c^2 = 2 ==> c = sqrt[(a^2 + b^2)/2]

Gary Litvin
www.skylit.com/calculus

---

[*] Assume you know the height of the original trapezoid. It may be any real number. Since *h* is a factor in all the area formulas it cancels out, but we need to pretend we know it.