CAS in High Schools Texts

Good afternoon and Thank you for coming.

According to the program my topic is “CAS in Algebra 1 through Calculus.” I’m not planning to tell you how to change your whole program so as to use CAS in the hour I have here today. Rather, I would like to share some thoughts, reflections, and ideas with you.

There are of course many CAS sessions at this meeting that will give you specific ideas on how to use your CAS to help your students better learn mathematics in high school. I will be more general; but I will look at a couple of interesting ways to use CAS in High School in order to illustrate some ideas.

CAS stands for Computer Algebra Systems – you knew that.

The “system” almost always includes everything that a scientific calculator can do (that is numerical computations), and everything a graphing calculators can do (draw graphs, make tables of values, write and run programs to do all these things). PLUS a CAS can “do algebra;” which to most people means it can manipulate variables in order to solve equations, and so forth.

I will limit my remarks to the "doing algebra" aspect of the CAS since that's what really differentiates them from graphing calculators (no pun intended).

First let me state my opinion that CAS should be in every students hands beginning in first year algebra *at the latest.* My reason for that is simple: they can learn more mathematics with a CAS than without.

Let's start then with the very basic question: "What is Algebra?"

- The Oxford Desk Dictionary say algebra is the "branch of mathematics that uses letter, etc. to represent numbers and quantities."

- The (online) American Heritage Dictionary say algebra is "A branch of mathematics in which symbols, usually letters of the alphabet, represent numbers or members of a specified set and are used to represent quantities and to express general relationships that hold for all members of the set."[1]

- Another online source says algebra is "branch of mathematics concerned with operations on sets of numbers or

other elements that are often represented by symbols. Algebra is a generalization of arithmetic and gains much of its power from dealing symbolically with elements and operations (such as addition and multiplication) and relationships (such as equality) connecting the elements."[2]

- And another: "A branch of mathematics marked chiefly by the use of symbols to represent numbers, as in the use of $a^2 + b^2 = c^2$ to express the Pythagorean Theorem."[3] This from the New Dictionary of Cultural Literacy – we should all be culturally literate.

- One more this time from Wikipedia: **Elementary algebra**, in which the properties of operations on the real number system are recorded using symbols as "place holders" to denote constants and variables, and the rules governing mathematical expressions and equations involving these symbols are studied."[4]

You get the idea – <u>algebra is symbols and symbol manipulation</u>.

But I never thought of algebra that way!

I knew that all that was part of algebra; but I don't think that's all it did. I'd prefer to think of algebra as a method for solving problems. I always thought of algebra as having 4 parts:

1. You begin with, for what of a better terms, I'll call a "problem" – some information that you need to know more about. Sometimes something specific and sometimes something more general. The "problem" is usually about something that concerned numbers in one way or another.

2. Next the problem is expressed in terms of variables and numbers and symbols telling you how they were related. (That's what makes an algebra problem different from your personal problems, for example.) Sometimes the "problem" starts at this step – already expressed in variables.

3. These symbols were manipulated according to certain rules to arrive at an "answer."

4. The answer was then compared to the original problem to see how well it worked. To see if it is a solution.

Apparently, according to the definitions I just read, algebra is ONLY the third of those four things – the symbol manipulation part!

If so that's really good news – we can forget about all that "real world" stuff and just concentrate on teaching the manipulation the variables, and of course we won't *want* a CAS, because the goal would be to teach symbol manipulation.

Well no, we can't do that. If you like my definition then the really good news is that computer algebra systems can do that third step for us and lets us and our students concentrate on the real algebra in a real setting, concentrating on solving the problem, not just solving the equation.

I suggest then that we let the CAS do Part 3 and we and our students concentrate on Parts 1, 2 and 4.

I propose that it is important for students to be knowledgeable about symbols and why and how they are manipulated, but it is not important to be very good at manipulating symbols.

After all Newton wasn't even good at arithmetic.[5]

SHOW NEWTON PAGE

Before we look at some examples let me take a short side trip.

To be used to its full advantage CAS will require changes in how we think about mathematics and about the high school curriculum. This is not a new thing. The topics taught in school are not, contrary to popular opinion, set in stone. Here is a short list of changes I've seen in my teaching career. (Okay that's a long time!)

- Computing square roots by "the algorithm." No more.
- Which gave way to: looking square roots up in tables along with using simplified radical form to find the decimal values. No more.
- We no longer teach computations with logarithms (expand). No more.
- We do not teach looking up functions in trig (or log) tables No more.
- Interpolation in tables. No more.
- Degrees, minutes and seconds have given way to decimal degrees

- Newton's Method has been dropped from the AP calculus course description (expand)
- And rationalizing denominators is on its death bed.

In a few minutes I'll suggest some others things that can be dropped. (Stay tuned)

Here are some ways that I think CAS can be used. This is certainly not an exhaustive list.

I'll try to be very general, but the theme is always the same: Let the CAS do the grunt work.

1. Delving into topics that require – on the side so to speak – manipulations and computations. These manipulations and computations detract from the point. Example:
   Prime factorization. How much time do you spend teaching methods (or *a* method) of factoring integers and how much do you spend using the factorization to learn about the properties of numbers? How you get the factors is not important; how you use them is.
   GO TO TI-INTEACTIVE There are quite a few CAS that are available. The TI Voyage 200 is my favorite, but there's the

TI89; HP models. Casio Class Pad, Maple, Mathematica, and other. I'm sure some good open source software will be available soon. I'm using TI-Interactive today because it will be easy for you to see and the syntax is almost identical to the TI89.

Factor(100!)

Factor({24,18})

LCM(24,18)

Factor(ans)

GCD(24,18)

Factor (ans)

$\sqrt{50}$, $\sqrt{5000}$

2. Step-by-step solutions
    a. Equations
    b. Formulas
    c. Systems of Equations[6]
    d. Comment: This is not really an important use IMO, but it does help students learn the steps without worrying about the arithmetic and shows their mistakes at once.
3. Data on Symbol use
    a. a+a+a+a+a = 5a and a a a a a = a^5

b. Practice with symbols: Example: Use seq(to write {3, 6, 9, 1,2 15} as many different ways as you can .

c. Theory of equations

i. (x + a)(x + b)(x + c) …. And examine patterns in coefficients

ii. Binomial theorem

4. Complicated computations – CUBIC SYMMETRY IF TIME

5. One line programs

a. Analytic geometry's 5 formulas

1. Slope
2. **distance**
3. midpoint
4. equation of a line
5. angle between lines

b. Example 1 **Is ()()()() a parallelogram**?

c. Equation of a **HYPERBOLA**

d. Example 2 **Law of Sines/ Cosines** SET MODE (f9) TO DEGREES, SET RANGE FOR ANGLE DO sss, sas, asa

This last example gives us more things to add to our list of things that won't be around much longer.

- Law of Sines DISCUSS
- Law of Cosines
- Quadratic formula

Now before you go away thinking I live in some kind of ideal world, let me assure you I know the issues. Here they are some of them:

- Equity

  This is usually framed like this: since everyone (or everyone's school district) cannot afford a CAS it is unfair to let others use them; they will have an unfair advantage. Well in a way that's true; they will have an unfair advantage, but that, IMO is not the real equity issue.

  The real issue is if you can learn more math and learn the math better with a CAS, then it's unfair to deprive anyone of a CAS. Everyone should have one! And it's unfair to those who have a CAS to forbid them from learning with it.

- Standardized tests ARE the curriculum. As long a standardized test require by hand rationalization of denominators then damn it, we're gonna teach by hand

rationalization of denominators. Even if it's a waste of time and mind numbing.

- Teacher training. There will be resistance from teachers (beside those area listed above) because they too are intimidated by technology.

- What p&p is to be retained? Some must be. Perhaps enough for students to understand what the CAS is doing, but maybe not how it's doing it.

- How students should show their work done on a CAS. Copy it onto their paper. Turn in a computer output, show the teacher the screen. Needs to be decided.

- Mathematical "beauty." I assure you I think the Law of Sines is one of the most beautiful things in mathematics. Nothing like it. But ….

[1] The American Heritage® Dictionary of the English Language, Fourth Edition Copyright © 2004, 2000 by Houghton Mifflin Company. Published by Houghton Mifflin Company. All rights reserved.

[2] The Columbia Electronic Encyclopedia, Sixth Edition Copyright © 2003, Columbia University Press. Licensed from Columbia University Press. All rights reserved. www.cc.columbia.edu/cu/cup/

[3] The New Dictionary of Cultural Literacy, Third Edition Edited by E.D. Hirsch, Jr., Joseph F. Kett, and James Trefil. Copyright © 2002 by Houghton Mifflin Company. Published by Houghton Mifflin.

[4] www.wikipedia.com "Algebra"

[5] *Isaac Newton* by JamesGleick, Vintage Books, New York, © 2003, p. 37.

[6] "XXX" in *The Mathematics Teacher* October 2006.p. xxx