Limits and Continuity 1 – Types of discontinuities

Look for two things in this talk: The first is the names and graphical appearance of various types of discontinuities, the second is the use of the word "limit" and the *notation* that goes with it, to describe the discontinuities.

Continuous functions

Holes in the graph (removable discontinuities) $f(x)=\dfrac{x^3+x^2-2x}{x-1}$

Jump Discontinuities: $y=\dfrac{\sqrt{x^2-4x+4}}{x^3-2x^2+x-2}=\dfrac{|x-2|}{(x-2)(x^2+1)}$

Vertical asymptotes: $f(x)=\dfrac{3}{x-1}$, and $g(x)=\dfrac{3}{(x-1)^2}$

Oscillating discontinuities $y=\sin\left(\frac{1}{x}\right)$

Definition of *Continuity at a point*:

A function, f, is continuous at $x = a$ if, and only if, $\lim_{x \to a} f(x) = f(a)$ (and both of these numbers are finite).

Definition of *Continuity on an interval*:

A function is continuous on an interval if, and only if, it is continuous at every point in the interval.

Endpoint continuity:

A function is continuous at the endpoint of an interval if, and only if, the one-sided limit approaching the end point from inside the interval is equal to the value at the endpoint.

Thus for the interval $[a,b]$, the function is continuous at $x = a$ if, and only if $\lim_{x \to a+} f(x) = f(a)$ and continuous at $x = b$ if, and only if $\lim_{x \to b-} f(x) = f(b)$